The Luttinger liquid concept for interacting electrons in one dimension

2002 J. Phys.: Condens. Matter 14 12783

(http://iopscience.iop.org/0953-8984/14/48/317)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 12783–12791 PII: S0953-8984(02)54079-0

The Luttinger liquid concept for interacting electrons in one dimension

K Schönhammer

Institut für Theoretische Physik, Universität Göttingen, Bunsenstr. 9, D-37073 Göttingen, Germany

Received 30 September 2002
Published 22 November 2002
Online at stacks.iop.org/JPhysCM/14/12783

Abstract

The theoretical description of interacting fermions in one spatial dimension is simplified by the fact that the low-energy spectrum of noninteracting fermions is identical to the one for a harmonic chain. This allows us to describe interacting fermions as a system of coupled oscillators. The exact solution of the Tomonaga–Luttinger model is discussed and results for the momentum distribution and spectral functions are presented. The general Luttinger liquid phenomenology is briefly discussed and various attempts at the experimental verification of the theoretical predictions are examined. Extended defects in semiconductors might also be candidate systems.

1. Introduction

The low-temperature thermodynamic properties of simple metals can be qualitatively understood in terms of the simple Sommerfeld model [1], which treats the conduction electrons as noninteracting fermions in a box. Typical results are a specific heat *linear* in temperature and a constant spin susceptibility in qualitative agreement with experiments. Landau's *Fermi liquid* theory [2] rests on the assumption of *quasi-particles* which are in a one-to-one correspondence to noninteracting fermions. This leads to a linear specific heat and a constant spin susceptibility but involves *renormalized* quantities like the effective mass and quasi-particle interaction parameters [2, 3], which are difficult to calculate microscopically. The consistency of the approach was shown using perturbation theory to infinite order and more recently by renormalization group techniques [4].

The problem of interacting fermions simplifies in one dimension. In a pioneering paper [5] Tomonaga treated the case of a two-body interaction which is long ranged in real space. He showed that the low-energy excitations of the noninteracting as well as the interacting system can be described in terms of *noninteracting bosons* [6]. The important idea used to solve the case of interacting fermions was the observation that a long-range interaction in real space is short ranged in momentum space and therefore only particles and holes in the vicinity of the Fermi points are involved in the interacting ground state and states with a low excitation

energy. To obtain his results, Tomonaga *linearized* the energy dispersion around the two Fermi points $\pm k_F$. Luttinger [7] later used a model with strictly linear energy dispersions and presented the exact result for the mean occupation numbers. The complete solution for the Luttinger model was presented by Mattis and Lieb [8]. A very elegant method for calculating correlation functions for the model is the bosonization of the fermion field operator [9–12]. The exponents of the anomalous power-law decay of various correlation functions are determined by the *anomalous dimension*, which can be calculated explicitly for the Tomonaga–Luttinger (TL) model [7–9]. Haldane [10] made the important observation that the low-energy physics of the exactly solvable TL model provides the *generic* scenario for one-dimensional fermions with repulsive interactions. As in the Landau Fermi liquid picture [2] a few parameters completely determine the low-energy physics. Generally they are as difficult to calculate as the Landau parameters. In contrast to the higher-dimensional case there are additional exactly solvable models for which Haldane's *Luttinger liquid* scenario can be tested and the parameters determining the anomalous dimension can be calculated using the Bethe ansatz technique [13].

An important manifestation of Luttinger liquids is called *spin charge separation*, i.e. for low-energy excitations the charge and spin degrees are completely decoupled. This shows up, for example, in the spectral function of the one-particle Green function [14, 15] which largely determines the photoemission spectrum. Recent high-resolution photoemission experiments on quasi one-dimensional conductors have been interpreted as showing Luttinger liquid behaviour [16]. Another important aspect of LL behaviour concerns the peculiar modification of the electronic properties of an LL when a *single* impurity with an arbitrarily weak backscattering potential is present. For a spinless LL with a *repulsive* two-body interaction, a perturbative renormalization group calculation [17] shows that the backscattering potential is a *relevant* perturbation. At low energy scales, even for a weak impurity, physical observables behave as if the system is split into two semi-infinite chains. This leads to a conductance which vanishes with a power law in T at low temperatures [17].

In section 2 the TL model and the basic concepts of LL physics are presented without going into too much technical detail and attempts at an experimental verification are discussed in section 3.

2. The Tomonaga–Luttinger model

The first step in the understanding of interacting fermions in one dimension ($d = 1$) is to realize that noninteracting fermions have the same type of low-energy excitations as a harmonic chain [18]. The low-energy excitations determine, for example, the low-temperature specific heat. Debye's famous T^3 law for the lattice contribution of three-dimensional solids reads in $d = 1$

$$c_L^{\text{Debye}} = \frac{\pi}{3} k_B \left(\frac{k_B T}{\hbar c_s} \right), \tag{1}$$

where c_s is the sound velocity. At low temperatures the electronic contribution to the specific heat in the 'Fermi gas' approximation of Pauli is also linear in T and involves the density of states of the noninteracting electrons at the Fermi energy. This yields for spinless fermions in $d = 1$

$$c_L^{\text{Pauli}} = \frac{\pi}{3} k_B \left(\frac{k_B T}{\hbar v_F} \right), \tag{2}$$

where v_F is the Fermi velocity. With the replacement $c_s \leftrightarrow v_F$ the results are *identical*. This suggests that apart from a scale factor the (low-energy) excitation energies and the degeneracies in the two types of systems are identical.

When a two-body interaction between the fermions is switched on, the ground state is no longer the filled Fermi sea but it has admixtures of (multiple) particle–hole pair excitations. In order to simplify the problem Tomonaga studied the *high-density limit* where the range of the interaction is much larger than the interparticle distance, using *periodic* boundary conditions [5]. Then the Fourier transform $\tilde{v}(k)$ of the two-body interaction is nonzero only for values $|k| \leqslant k_c$ where the cut-off k_c is much smaller than the Fermi momentum $k_c \ll k_F$. This implies that for not too strong interactions the ground state and low-energy excited states have negligible admixtures of holes deep in the Fermi sea and particles with momenta $|k| - k_F \gg k_c$. In the two intermediate regions around the two Fermi points $\pm k_F$, with particle–hole pairs present, the dispersion ε_k is *linearized* in order to apply Bloch's 'sound wave method' [18]

$$k \approx \pm k_F: \qquad \varepsilon_k = \varepsilon_F \pm v_F(k \mp k_F). \tag{3}$$

Tomonaga realized that the Fourier components of the operator of the density

$$\hat{\rho}_n = \int_{-L/2}^{L/2} \hat{\rho}(x) \mathrm{e}^{-\mathrm{i}k_n x}\, \mathrm{d}x = \sum_{n'} c^\dagger_{n'} c_{n'+n}, \tag{4}$$

where $c^\dagger_n(c_n)$ creates (annihilates) a fermion in the state with momentum $k_n = \frac{2\pi}{L}n$, plays a central role in the interaction term, as well as the kinetic energy. Apart from additional terms involving the particle number operator the two-body interaction is given by

$$\hat{V} = \frac{1}{2L} \sum_n \tilde{v}(k_n) \hat{\rho}_n \hat{\rho}_{-n}. \tag{5}$$

Tomonaga's important step was to split $\hat{\rho}_n$ for $|k_n| \ll k_F$ into two parts, one containing operators of *'right movers'*, i.e. involving fermions near the right Fermi point k_F with velocity v_F, and *'left movers'*, involving fermions near $-k_F$ with velocity $-v_F$

$$\hat{\rho}_n = \sum_{n' \geqslant 0} c^\dagger_{n'} c_{n'+n} + \sum_{n'<0} c^\dagger_{n'} c_{n'+n} \equiv \hat{\rho}_{n,+} + \hat{\rho}_{n,-} \tag{6}$$

where the details of the splitting for small $|n'|$ are irrelevant. Their commutation relations in the *low-energy subspace* with no holes deep in the Fermi sea are [5]

$$[\hat{\rho}_{m,\alpha}, \hat{\rho}_{n,\beta}] = \alpha m \delta_{\alpha\beta} \delta_{m,-n} \hat{1}. \tag{7}$$

If one defines the operators

$$b_n \equiv \frac{1}{\sqrt{|n|}} \begin{cases} \hat{\rho}_{n,+} & \text{for } n > 0 \\ \hat{\rho}_{n,-} & \text{for } n < 0 \end{cases} \tag{8}$$

and the corresponding adjoint operators $b^\dagger_n$ this leads, using $\rho^\dagger_{n,\pm} = \rho_{-n,\pm}$, to the bosonic commutation relations

$$[b_n, b_m] = 0, \qquad [b_n, b^\dagger_m] = \delta_{mn} \hat{1}. \tag{9}$$

As $\hat{V}$ is bilinear in the $\hat{\rho}_n$ the same is true for the $\hat{\rho}_{n,\pm}$. As the additional important step in solving the interacting model Tomonaga used Bloch's insight, that the *kinetic energy* can also be expressed in terms of the Bose operators [5]. Therefore, apart from an additional term containing particle number operators, *the Hamiltonian for the interacting fermions is a quadratic form in the boson operators*

$$H = \sum_{n>0} \hbar k_n \left\{ \left(v_F + \frac{\tilde{v}(k_n)}{2\pi\hbar} \right) (b^\dagger_n b_n + b^\dagger_{-n} b_{-n}) + \frac{\tilde{v}(k_n)}{2\pi\hbar} (b^\dagger_n b^\dagger_{-n} + b_{-n} b_n) \right\}$$
$$+ \frac{\hbar\pi}{2L} [v_N \mathcal{N}^2 + v_J \mathcal{J}^2] \equiv H_B + H_{\mathcal{N},\mathcal{J}}, \tag{10}$$

where $\mathcal{N} \equiv \mathcal{N}_+ + \mathcal{N}_-$ is the total particle number operator, $\mathcal{J} \equiv \mathcal{N}_+ - \mathcal{N}_-$ the 'current operator', and the velocities are given by $v_N = v_F + \tilde{v}(0)/\pi\hbar$ and $v_J = v_F$. Here v_N determines the energy change for adding particles without generating bosons while v_J enters the energy change when the difference in the number of right and left movers is changed. As the particle number operators $\mathcal{N}_\pm$ commute with the boson operators $b_m(b_m^\dagger)$ the two terms H_B and $H_{\mathcal{N},\mathcal{J}}$ in the Hamiltonian *commute* and can be treated separately. Because of the translational invariance, the two-body interaction only couples the modes described by $b_n^\dagger$ and b_{-n}. With the Bogoliubov transformation $\alpha_n^\dagger = b_n^\dagger \cosh\theta_n - b_{-n} \sinh\theta_n$ the Hamiltonian H_B can be brought into the form

$$H_B = \sum_{n\neq 0} \hbar\omega_n \alpha_n^\dagger \alpha_n + \text{constant}, \tag{11}$$

where the $\omega_n = v_F|k_n|\sqrt{1+\tilde{v}(k_n)/\pi\hbar v_F}$ follow from 2×2 eigenvalue problems corresponding to the condition $[H_B, \alpha_n^\dagger] = \omega_n \alpha_n^\dagger$. For small k_n one obtains for smooth potentials $\tilde{v}(k)$ again a *linear* dispersion $\omega_n \approx v_c|k_n|$, with the *'charge velocity'* $v_c = \sqrt{v_N v_J}$, which is larger than v_F for $\tilde{v}(0) > 0$. The parameter θ_n in the Bogoliubov transformation is determined by

$$\tanh(2\theta_n) = -\frac{\tilde{v}(k_n)}{2\pi\hbar v_F + \tilde{v}(k_n)}. \tag{12}$$

For fixed particle numbers N_+ and N_-, the excitation energies of the *interacting* Fermi system are given by $\sum_m \hbar\omega_m n_m$ with integer occupation numbers $0 \leqslant n_m < \infty$. For small enough excitation energies the only difference between the excitation spectrum for fixed particle numbers and that for the noninteracting case is the replacement $v_F \leftrightarrow v_c$.

The interacting ground state with its multiple particle–hole pairs above the Fermi sea is given by [10]

$$|E_0(N)\rangle \sim \exp\left(\sum_{n>0} \tanh(\theta_n) b_n^\dagger b_{-n}^\dagger\right)|F(N)\rangle. \tag{13}$$

In order to elucidate the *non-Fermi liquid* character of the TL model with $\tilde{v}(0) \neq 0$ it is best to study the dynamics of states $c_{k_n}^{(\dagger)}|E_0(N)\rangle$ with an additional particle (hole). Only in the noninteracting limit are these states eigenstates of the Hamiltonian and therefore have an infinite lifetime. In the (three-dimensional) Fermi liquid theory the *quasi-particle(hole)* concept is very successful, with the lifetime of these states going to infinity when the momentum approaches the Fermi surface. If one defines the 'quasi-hole weight' Z_F as

$$Z_F \equiv |\langle E_0^{(-k_F)}(N-1)|c_{k_F}|E_0(N)\rangle|^2, \tag{14}$$

where $|E_0^{(-k_F)}(N-1)\rangle$ is the $(N-1)$-particle groundstate with total momentum $-k_F$, one has $Z_F = 1$ for noninteracting fermions. Assuming the validity of Fermi liquid theory one expects $Z_F \to$ constant in the limit $L \to \infty$. An exact calculation of Z_F for the TL model is possible by 'bosonizing' the field operators c_k [9–12] as well. This yields

$$Z_F = \exp\left(-\sum_{n>0} \frac{2\sinh^2(\theta_n)}{n}\right). \tag{15}$$

In the large L limit the sum can be converted to an integral and one obtains $Z_F \sim (1/L)^\alpha$, where

$$\alpha = 2\sinh^2[\theta(0)] = \left[1 - \left(\frac{\tilde{v}(0)}{2\pi\hbar v_F + \tilde{v}(0)}\right)^2\right]^{-1/2} - 1, \tag{16}$$

is called the *anomalous dimension*, as α determines the anomalously slow spatial decay of the one-particle Green function. If one defines the 'stiffness constant' $K \equiv \mathrm{e}^{2\theta(0)} = (v_J/v_N)^{1/2}$

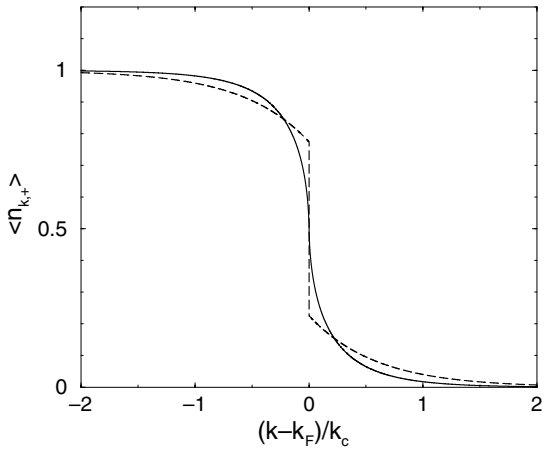

Figure 1. The full curve shows the average occupation $\langle n_{k,+}\rangle$ for a TL model with $\alpha = 0.6$. The dashed curve shows the expectation from Fermi liquid theory, where the discontinuity at k_F is given by Z_F. This can also be realized in a TL model with $\tilde{v}(0) = 0$. The details of the interaction are specified in the text.

the anomalous dimension can also be written as $\alpha = (K-1)^2/2K$. In contrast to Fermi liquid theory the quasi-hole weight Z_F vanishes in a power-law fashion for $L \to \infty$, if $\tilde{v}(0)$ is different from zero.

The appearance of power laws in the TL model was first realized by Luttinger [7]. He found that the average occupation $\langle n_{k,+}\rangle \equiv \langle E_0(N)|c_k^\dagger c_k|E_0(N)\rangle$ in the interacting ground state for $k \approx k_F$ behaves as

$$\langle n_{k,+}\rangle - \frac{1}{2} \sim \left|\frac{k-k_F}{k_c}\right|^\alpha \mathrm{sign}(k_F - k). \tag{17}$$

This is shown in figure 1 in comparison with Fermi liquid theory. The full line was calculated assuming $\sinh^2[\theta(k)] = 0.3\mathrm{e}^{-2|k|/k_c}$, while the dashed line corresponds to $\sinh^2[\theta(k)] = 0.6(|k|/k_c)\mathrm{e}^{-2|k|/k_c}$.

Electrons are spin one-half particles and for their description it is necessary to include the spin degree of freedom in the model. For a fixed quantization axis the two spin states are denoted by $\sigma = \uparrow, \downarrow$. The fermionic creation (annihilation) operators carry an additional spin label as well as the $\hat{\rho}_{n,\pm,\sigma}$ and the boson operators $b_{n,\sigma}$ which in a straightforward way generalize equation (8). It is useful to switch to new boson operators $b_{n,a}$ with $a = c, s$

$$\begin{aligned} b_{n,c} &\equiv \frac{1}{\sqrt{2}}(b_{n,\uparrow} + b_{n,\downarrow}) \\ b_{n,s} &\equiv \frac{1}{\sqrt{2}}(b_{n\uparrow} - b_{n,\downarrow}), \end{aligned} \tag{18}$$

which obey $[b_{a,n}, b_{a',n'}] = 0$ and $[b_{a,n}, b_{a',n'}^\dagger] = \delta_{aa'}\delta_{nn'}\hat{1}$. The kinetic energy can be expressed in terms of the 'charge' (c) and 'spin' (s) boson operators using $b_{n,\uparrow}^\dagger b_{n,\uparrow} + b_{n\downarrow}^\dagger b_{n\downarrow} = b_{n,c}^\dagger b_{n,c} + b_{n,s}^\dagger b_{n,s}$. If one defines the interaction matrix elements $\tilde{v}_c(q) \equiv 2\tilde{v}(q)$ and $v_s(q) = 0$, $\mathcal{N}_{\pm,c} \equiv (\mathcal{N}_{\pm,\uparrow} + \mathcal{N}_{\pm,\downarrow})/\sqrt{2}$ and $\mathcal{N}_{\pm,s}$ as the corresponding difference, one can write the TL Hamiltonian $\tilde{H}_{TL}^{(1/2)}$ for spin one-half fermions as

$$\tilde{H}_{TL}^{(1/2)} = \tilde{H}_{TL,c} + \tilde{H}_{TL,s}, \tag{19}$$

where the $\tilde{H}_{TL,a}$ are of the form equation (10) but the interaction matrix elements have the additional label a. The two terms on the rhs of equation (19) *commute*, i.e. the 'charge' and 'spin' excitation are completely independent. This is usually called 'spin–charge separation'. The diagonalization of the two separate parts proceeds exactly as before and the low-energy excitations are 'massless bosons' $\omega_{n,a} \approx v_a|k_n|$ with *charge velocity* $v_c = (v_{J_c} v_{N_c})^{1/2}$ and *spin velocity* $v_s = (v_{J_s} v_{N_s})^{1/2} = v_F$. The corresponding two stiffness constants are given by $K_c = (v_{J_c}/v_{N_c})^{1/2}$ and $K_s = 1$. If one allows the coupling constants in equation (10) in front of the $b^\dagger b$ and the $b^\dagger b^\dagger$ terms to be different, the spin velocity v_s differs from v_F. If in addition the interaction is allowed not to be spin rotational invariant, K_s differs from 1 [19].

The low-temperature thermodynamic properties of the TL model including spin can be expressed in terms of the four quantities v_c, K_c, v_s and K_s. As an example we consider the specific heat. Due to spin–charge separation it has two additive contributions of the same form as in equations (1) and (2). If we denote, as usual, the proportionality factor in the linear T term by γ one obtains

$$\frac{\gamma}{\gamma_0} = \frac{1}{2}\left(\frac{v_F}{v_c} + \frac{v_F}{v_s}\right), \tag{20}$$

where γ_0 is the value in the noninteracting limit.

A simple manifestation of spin–charge separation occurs in the time evolution of a localized perturbation of, for example, the spin-up density. If the initial state of the system involves a perturbation of *right movers* only, and the perturbation is sufficiently smooth, the initial perturbation is split into four parts which move with velocities $\pm v_c$ and $\pm v_s$ without changing the initial *shape*. If only the initial expectation values of the $b_{n,\uparrow}$ are different from zero one obtains for $\delta\langle\rho_\uparrow(x,0)\rangle \equiv F(x)$ using the simple time evolution $\alpha_{n,a}(t) = \alpha_{n,a}\mathrm{e}^{-\mathrm{i}\omega_{n,a}t}$ for $a = c, s$

$$\delta\langle\rho_\uparrow(x,t)\rangle = \sum_a \left[\frac{1+K_a}{4}F(x - v_a t) + \frac{1-K_a}{4}F(x + v_a t)\right]. \tag{21}$$

For the spin rotational invariant case $K_s = 1$ there is no contribution which moves to the left with the spin velocity. The following comment should be made: spin–charge separation is often described as the fact that when an electron is injected into the system *its* spin and charge move independently with different velocities. This is very misleading as it is a *collective* effect of the total system which produces expectation values as in equation (21).

The easiest way to understand the important manifestation of spin–charge separation in the momentum resolved one-particle spectral functions [14, 15] is to make use of the bosonization of the electronic field operators. These spectral functions which are relevant for the description of angular resolved photoemission are just the spectral resolutions of the hole states $c_{k,\sigma}|E_0(N)\rangle$ discussed earlier

$$\rho^<_\sigma(k,\omega) = \sum_j |\langle E_j(N-1)|c_{k,\sigma}|E_0(N)\rangle|^2 \delta(\omega + E_j(N-1) - E_0(N-1)). \tag{22}$$

The k-resolved spectral functions show a drastic difference from the model without spin. The delta peaks of the noninteracting model are broadened into *one* power-law threshold in the model without spin and *two* power-law singularities (see figure 2) in the model including spin [14, 15]. The 'peaks' disperse *linearly* with $k - k_F$.

For the momentum integrated spectral functions, relevant for angular integrated photoemission, one obtains $\rho_{\alpha,\sigma}(\omega) \sim |\omega|^\alpha$ as in the spinless model. For the calculation of other correlation functions we refer to recent reviews [19, 20].

Integrable lattice models like the 1D Hubbard model can be shown to be Luttinger liquids (except at half filling where low-energy umklapp processes lead to the opening of a Mott–

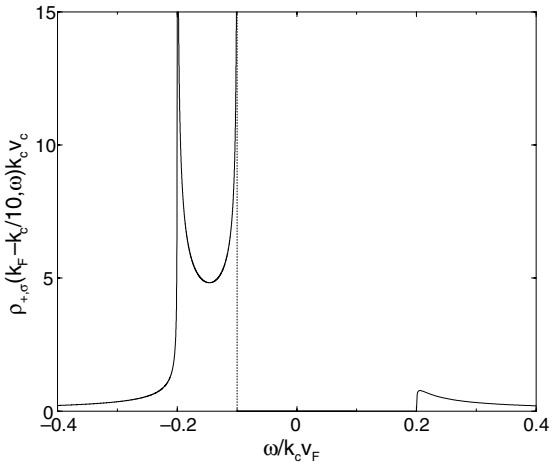

Figure 2. Spectral function $\rho_{+,\sigma}(k_F + \tilde{k}, \omega)$ as a function of normalized frequency for $\tilde{k} = -k_c/10$ for the TL model with a spin-independent interaction. The parameters are chosen such that $v_c = 2v_F$ and $\alpha = 1/8$. In the noninteracting limit there is a delta peak at -0.1.

Hubbard charge gap) and the Luttinger liquid parameters v_c, v_s and K_c can be calculated exactly [13].

3. On the experimental verification of LL behaviour

Strictly 1D systems are a theoretical idealization. Apart from this even the coupling to an experimental probe presents a nontrivial disturbance of a Luttinger liquid. Unfortunately the weak coupling of a 1D system to such a probe as well as the coupling between several LLs is not completely understood theoretically [19]. The coupling between the chains in a very anisotropic 3D compound generally, at low enough temperatures, leads to true *long-range order*. The order develops in the phase for which the algebraic decay of the corresponding correlation function of the single-chain LL is the slowest [19]. This can lead, for example, to charge density wave (CDW) order, spin density wave (SDW) order or superconductivity.

There exist several types of experimental system where a predominantly 1D character can be hoped to lead to an (approximate) verification of the physics of Luttinger liquids in an appropriate temperature and energy window. In the following we present a short list of the most promising systems and discuss some of the experimental techniques which have been used.

The following *systems* look promising:

- Highly anisotropic 'quasi-one-dimensional' conductors. There has been extensive work on organic conductors like the Bechgaard salts [16, 21], as well as inorganic materials [22].
- Artificial quantum wires. Two important types of realizations are quantum wires in *semiconductor heterostructures* [23, 24] or quantum wires on *surface substrates* [25, 26].
- Carbon nanotubes. The long cylindrical fullerenes called quantum nanotubes are also quantum wires but have been listed separately because of their special importance in possible future applications like 'molecular electronics' [27, 28]. Using the peculiar band structure of the π-electrons of a single graphite plane it was shown that single wall 'armchair' nanotubes should show LL behaviour with $K_c \sim 0.2$–0.3 down to very low temperatures [29, 30], despite the fact that *two* low energy channels are present.

- Fractional quantum Hall fluids. Electrons at the edges of a two-dimensional fractional quantum Hall system can be described as a *chiral Luttinger liquid* [31]. The power-law tunnelling density of states observable in the tunnelling current–voltage characteristics shows power laws of extraordinary quality [32]. As in these chiral LLs the right- and left-movers are *spatially separated* the edge state transport is quite different from the case of quantum wires.
- Extended defects in semiconductors might be another possible realization [33].

Promising *experimental techniques* for verifying LL behaviour are:

- High-resolution photoemission. One of the earliest claims of possible verification of Luttinger liquid behaviour was from angular integrated photoemission of the Bechgaard salt $(TMTSF)_2PF_6$, which showed a power-law supression at the chemical potential with an exponent of order 1 over an energy range of almost 1 eV [34]. As there are doubts that this suppression can be simply explained by the LL power-law behaviour a large number of other quasi-one-dimensional conductors were examined [16, 22, 25, 26, 35]. All these studies indicate that an unambiguous identification of spin–charge separation in a 1D metal by photoemission is still lacking.
- Transport. As discussed in the introduction even a single impurity has a drastic effect on the conductance of a LL, which vanishes as a power law with temperature. Another issue is the 'conductance puzzle' of a clean LL. There has been an extended discussion about whether the quantized value e^2/h for noninteracting electrons in a single channel is modified by the interaction to $K_c e^2/h$ [36, 37]. Apparently the answer depends sensitively on the assumptions made about the contacts, a very delicate theoretical as well as experimental problem [38]. Experimental results are available for cleaved edge overgrowth quantum wires [23] as well as carbon nanotubes [39–41]. In the nanotubes the authors observe approximate power laws of the conductance which seem to be consistent with LL behaviour.
- Optical properties. The optical behaviour of different Bechgaard salts was analysed recently using LL concepts [42].

Obviously neither the list of systems nor that of methods is close to being complete. They were presented to show that intensive experimental activities are being pursued in the attempt to verify the elegant LL concept put forward by theoreticians. Further work on both sides is necessary to come to unambiguous conclusions.

References

[1] Ashcroft N W and Mermin N D 1976 *Solid State Physics* (New York: Holt, Rinehart and Winston)
[2] Landau L D 1957 *Sov. Phys.–JETP* **3** 920
[3] Pines D and Nozieres P 1966 *The Theory of Quantum Liquids* (New York: Benjamin)
[4] For a review see
Shankar R 1994 *Rev. Mod. Phys.* **66** 129
[5] Tomonaga S 1950 *Prog. Theor. Phys.* **5** 544–69
[6] Reprints of many of the important papers in this field together with an excellent introduction are presented in Stone M (ed) 1994 *Bosonization* (Singapore: World Scientific)
[7] Luttinger J M 1963 *J. Math. Phys.* **4** 1154
[8] Mattis D M and Lieb E H 1965 *J. Math. Phys.* **6** 304
[9] Luther A and Peschel I 1974 *Phys. Rev.* B **9** 2911
[10] Haldane F D M 1981 *J. Phys. C: Solid State Phys.* **14** 2585
[11] Gogolin A O, Nersesyan A A and Tsvelik A M 1998 *Bosonization and Strongly Correlated Systems* (Cambridge: Cambridge University Press)
[12] v Delft J and Schoeller H 1998 *Ann. Phys., Lpz.* **7** 225

[13] Schulz H J 1990 *Phys. Rev. Lett.* **64** 2831
[14] Meden V and Schönhammer K 1992 *Phys. Rev.* B **46** 15753
[15] Voit J 1993 *Phys. Rev.* B **47** 6740
[16] For a review see
Grioni M and Voit J 2000 High-resolution photoemission studies of low-dimensional system *Electron Spectroscopies Applied to Low-Dimensional Materials* ed H Stanberg and H Hughes (Dordrecht: Kluwer) p 501
[17] Kane C L and Fisher M P A 1992 *Phys. Rev. Lett.* **68** 1220
Kane C L and Fisher M P A 1992 *Phys. Rev.* B **46** 15233
[18] Bloch F 1934 *Helv. Phys. Acta* **7** 385
[19] Voit J 1995 *Rep. Prog. Phys.* **58** 977–1116
[20] Schönhammer K 2003 Luttinger liquids: the basic concepts *Strong Interactions in Low Dimensions* ed D Baeriswyl and L Degiorgi (Dordrecht: Kluwer)
[21] Jerome D and Schulz H J 1982 *Adv. Phys.* **31** 299
[22] Gweon G H, Denlinger J D, Allen J W, Claessen R, Olson C G, Höchst H, Marcus J, Schlenker C and Schneemeyer L F 2001 *J. Electron Spectrosc. Relat. Phenom.* **117–118** 481
[23] de Picciotto R, Störmer H L, Pfeiffer L N, Baldwin K W and West K W 1999 *Nature* **397** 598
[24] Auslaender O M, Yacobi A, de Piciotto R, Baldwin K W, Pfeiffer L N and West K W 2002 *Science* **295** 825
[25] Segovia P, Purdie D, Hengsberger M and Baer Y 1999 *Nature* **402** 504
[26] Losio R, Altmann K N, Kirakosian A, Lin J L, Petrovykh D Y and Himpsel F J 2001 *Phys. Rev. Lett.* **86** 4632
[27] Saitoh R, Dresselhaus G and Dresselhaus M S 1998 *Physical Properties of Carbon Nanotubes* (UK: Imperial College Press)
[28] Dekker C 1999 *Phys. Today* **52** 22
[29] Egger R and Gogolin A 1997 *Phys. Rev. Lett.* **79** 5082
Egger R and Gogolin A 1998 *Eur. Phys. J.* B **3** 281
[30] Kane C L, Balents L and Fisher P A M 1997 *Phys. Rev. Lett.* **79** 5086
[31] Wen X G 1991 *Phys. Rev.* B **43** 11025
[32] Chang A M, Pfeiffer L N and West K W 1998 *Phys. Rev. Lett.* **77** 2538
[33] Schröter W and Cerva H 2002 *Defect Interaction and Clustering in Semiconductors* ed S Pizzini (Switzerland: Scitec Publications)
[34] Dardel B, Malterre D, Grioni M, Weibel P, Baer Y, Voit J and Jerome D 1993 *Europhys. Lett.* **24** 687
[35] Claessen R, Singh M, Schwingschlögl U, Blaha P, Dressel M and Jacobsen C S 2002 *Phys. Rev. Lett.* **88** 096402
[36] Maslow D L and Stone M 1995 *Phys. Rev.* B **52** R5539
[37] Safi I and Schulz H J 1995 *Phys. Rev.* B **52** R17040
[38] Egger R and Grabert H 1998 *Phys. Rev.* B **58** 10761
[39] Bockrath M, Cobden D H, Lu J, Rinzler A G, Smalley R E, Balents L and McEuen P L 1999 *Nature* **397** 598
[40] Yao Z, Postma H, Balents L and Dekker C 1999 *Nature* **402** 273
[41] Postma H, Teepen T, Yao Z, Grifoni M and Dekker C 2000 *Science* **293** 76
[42] Schwarz A, Dressel M, Grüner G, Vescoli V, Degiorgi L and Giamarchi T 1998 *Phys. Rev.* B **58** 1261